NAVAL POSTGRADUATE SCHOOL
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THESIS

RECIPROCITY CALIBRATION OF AN UNDERWATER
TRANSDUCER BY THE DELTA-Z METHOD

by

Raynald Bédard

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Thesis advisor: S.R. Baker

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Reciprocity Calibration of an Underwater Transducer by the Delta-Z Method

by

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Major, Canadian Forces
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Submitted in partial fulfillment of the
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ABSTRACT

A method for determining the free-field open-circuit voltage sensitivity of a reversible underwater electroacoustic transducer from the difference in its input electrical impedance when loaded by water and air was investigated theoretically and experimentally. An equation for the sensitivity was derived using reciprocal two-port network theory. The theory takes into account the diffraction due to the finite size of the transducer, its finite mechanical impedance, and its free-field radiation impedance in water. An experiment to test the predictions of the theory was performed using a 6" diameter, hollow, piezoelectric ceramic spherical transducer. The result of the experiment agreed within several dB with a standard comparison calibration over the frequency range for which both the electrical impedance and comparison calibration data are considered reliable. The calibration method described, which has been termed the Delta-Z Method, may be useful for *in-situ* monitoring of transducer sensitivity in installations which can be flooded and purged.

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I. INTRODUCTION

A. BACKGROUND

For the purpose of the research described in this thesis, an acoustical tranducer is a device which transforms fluctuations of environmental pressure into electrical quantities or vice-versa. A reversible acoustical transducer is one which can act as either a transmitter or a receiver.

There exist many calibration techniques which allow one to determine a transducer's receiving sensitivity, that is, the electrical output for a given pressure field at the transducer face, and/or its transmitting response, that is, the pressure field resulting from a given electrical input. The focus of this study is on a method introduced by the Royal Australian Navy (RAN) whereby the open circuit voltage sensitivity of a reversible underwater transducer can be obtained from the difference in its input electrical impedance (ΔZ) when loaded by water and air. This method shall hereafter be referred to as the Delta-Z method.

While there is agreement that the sensitivity can be obtained from ΔZ, there is disagreement over the appropriate analysis of the Delta-Z method and consequently over the formula which applies [Ref. 1]. Preliminary investigations of the Delta-Z method applied to in-service transducers were made by Taylor [Ref. 1] and Westbrook [Ref. 2]; however the experimental results were inconclusive.

Because of its experimental simplicity, the Delta-Z method has great potential for *in-situ* transducer calibration. For this reason it was chosen

for further study. It was hoped that, by using a simple geometry transducer, for which many of the needed parameters can be easily calculated, the controversy over the proper analysis of the method could be resolved and its usefulness established.

B. OBJECTIVE

The objective of the research described in this thesis is to develop, test and evaluate an underwater transducer calibration procedure for a simple geometry reversible transducer based on the difference in its input electrical impedance when loaded by water and air.

C. OUTLINE OF REMAINDER OF THESIS

In Chapter II the various standard methods used for transducer calibration and the theory associated with the Delta-Z method are discussed. The experimental procedure and the raw data gathered from the spherical transducer are described in Chapter III, while the analysis of this data and its associated results are part of Chapter IV. A discussion of the results follows in Chapter V. A summary and some conclusions and recommendations appear in Chapter VI.

II. THEORY

A. STANDARD CALIBRATION METHODS

The calibration methods described in this section are discussed in greater detail by Bobber [Ref. 3] and some portions have been extracted from his book.

1. Comparison Calibration method

In this procedure the electrical output voltage of the transducer under test is compared of to that of a standard transducer for which the sensitivity is known. It is the most expedient and widely used method to determine the absolute sensitivity of most transducers. The results obtained using this method will be used throughout this thesis as the standard to which the results of other methods will be compared.

The open-circuit output voltage, e_S, of the standard hydrophone is measured in a free-field position with its acoustic axis pointed towards the projector. The standard hydrophone is replaced by the unknown hydrophone and the open-circuit output voltage, e_X, is measured. If the free-field voltage sensitivity of the standard is M_S, then the sensitivity of the unknown is found from:

$$M_X = \frac{M_S\, e_X}{e_S} \qquad (2.1)$$

2. Conventional Reciprocity Calibration Method

A conventional reciprocity calibration requires three transducers of which one serves only as a projector (P), one is a reciprocal transducer (T) and serves as both a projector and a hydrophone, and one serves only as a hydrophone (H). Any one of the three transducers can be the unknown or the one being calibrated; however, the calibration formula usually is derived for the free-field voltage sensitivity, M_H, of the hydrophone. The measurements are made in the far-field under free-field conditions.

The arrangements and measurements made are shown schematically in Figure 1 and the procedure is as follows:

a. the input current, i_P, to the projector is measured while the hydrophone free-field output voltage, e_{PH}, is also measured ,

b. the same process is repeated with the hydrophone replaced by the transducer and its output voltage, e_{PT}, is measured,

c. the projector is then replaced by the transducer and both the input current, i_T, and the output voltage of the hydrophone, e_{TH}, are measured.

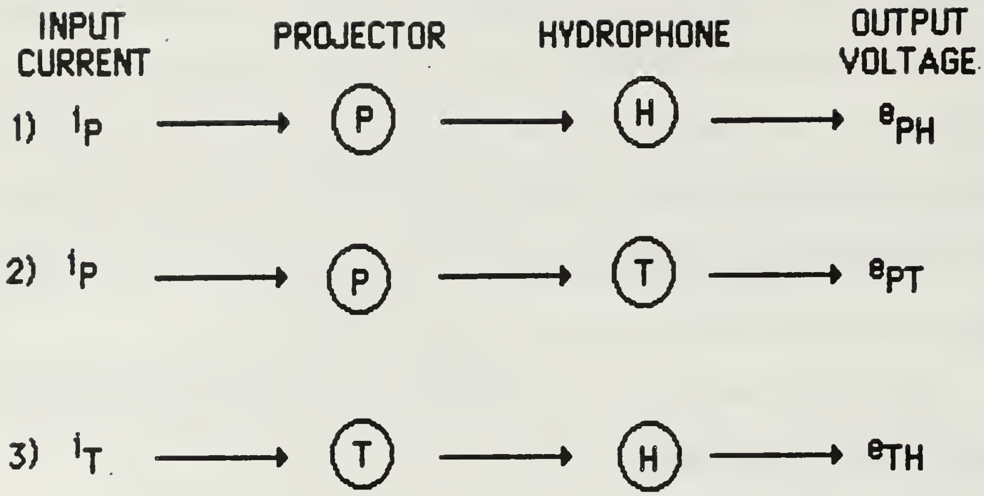

Figure 1. Diagram of the Measurements Needed for a Conventional Reciprocity Calibration.

The free-field open-circuit voltage sensitivity, M_H, of the hydrophone is calculated using the following equation:

$$M_H = \left[\frac{e_{PH} e_{TH} J}{e_{PT} i_T}\right]^{1/2} \qquad (2.2)$$

where J is the spherical reciprocity parameter:

$$J = \frac{2d}{\rho f}.$$

In this case, d is the distance between the acoustic centers of the two transducers, ρ is the density of the medium and f is the frequency of operation.

3. Two-transducer Reciprocity

Only a single transmitting current and receiving voltage measurement (i_T and e_{TH}) is required to perform a reciprocity calibration if two identical transducers are employed. In this case equation 2.2 is replaced by

$$M_H = \left[\frac{e_{TH}\,J}{i_T}\right]^{1/2} \tag{2.3}$$

B. SELF-RECIPROCITY METHODS

1. Surface Reflected Self-reciprocity Method

A single transducer can be used in a two-transducer reciprocity calibration procedure if it is placed near a perfectly reflecting surface; reflecting a transmitted acoustic pulse back to the transducer, it receives its own transmission. This self-reciprocity arrangement is described by Bobber [Ref. 3] and is shown schematically in Figure 2 where the air-water interface is used as the perfectly reflecting surface.

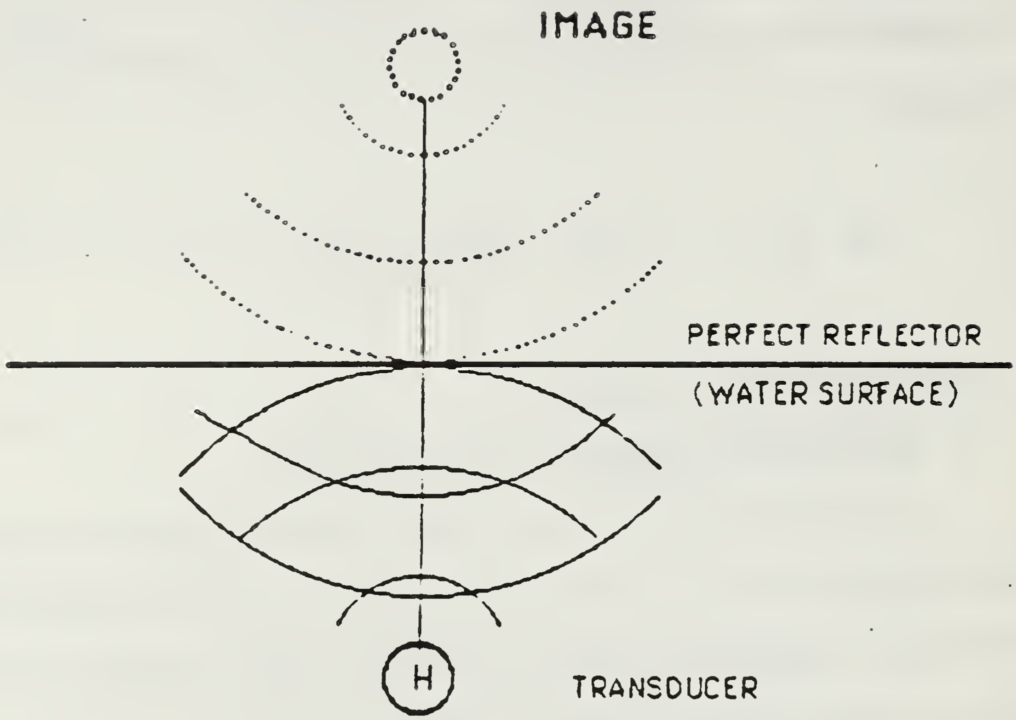

Figure 2. Self-reciprocity Arrangement

The resulting free-field open-circuit voltage sensitivity of the transducer, M_H, is determined by the expression:

$$M_H = \left[\frac{e_{TH} J_s}{i_T}\right]^{1/2} \qquad (2.4)$$

where:

e_{TH} = open-circuit voltage across terminals during receipt of the reflected signal,

i_T = current flowing into the transducer during transmission,

J_S = spherical reciprocity parameter:

$$J_S = \frac{2d}{\rho f} \tag{2.5}$$

d = twice the distance from the radiating face of the transducer to the reflecting surface,

ρ = the fluid mass density,

f = frequency of the transmitted signal.

2. Sabin's Delta-Z Method

Sabin [Ref. 4] pointed out that the ratio e_{th}/i_t was equal to the difference the input electrical impedance before and after receipt of the reflected pulse:

$$\Delta Z = Z_T - Z_f = \frac{e_{TH}}{i_T}, \tag{2.6}$$

where

Z_T = input electrical impedance after receipt of the reflected pulse,

Z_f = input electrical impedance before receipt of the reflected pulse, i.e. in a free-field. The expression for the open-circuit sensitivity from equation 2.4 then becomes

$$M_H = (\Delta Z J_S)^{1/2} \tag{2..7}$$

3. Royal Australian Navy Method

The Royal Australian Navy (RAN) developed a calibration method, claimed to be an extension of Sabin's method, in which the depth of water for the reflected impedance measurement is taken to be zero (the transducer is in air) [Ref. 5]. The planar reciprocity parameter is used to determine the sensitivity instead of the spherical reciprocity parameter.

The electrical impedance of the transducer to be calibrated is measured both in water (Z_{wet}) and in air (Z_{dry}) where:

$$\Delta Z = Z_{wet} - Z_{dry}, \qquad (2.8)$$

so that the open-circuit sensitivity now becomes:

$$M_H = (\Delta Z \, J_p)^{1/2} \qquad (2.9)$$

where:

J_p = the planar reciprocity parameter,

$$J_p = \frac{2A}{\rho c}, \qquad (2.10)$$

A = area of the active face of the transducer,

ρ = density of the medium,

c = speed of sound propagation in the medium, and

M_H = free-field voltage sensitivity.

The method described above will be hereafter referred to as the Australian or RAN method, while the method that will be developed in the following section will be referred to as the Delta-Z method.

C. RECIPROCAL TWO-PORT NETWORK ANALYSIS OF THE DELTA-Z METHOD

The treatment of the Delta-Z method as an extension of Sabin's method is not justified. For one thing, in conventional reciprocity methods each transducer is assumed to be in the far field of the other. Applied to reflection methods, this means that the transducer must be in the far field of its image in the reflecting surface. This condition cannot be met for a finite size transducer as the distance to that surface approaches zero. Furthermore, as will be apparent from the theory developed in this section and in the experimental results presented in Chapter V, the appearance of the planar reciprocity parameter in equation 2.9 is incorrect (notwithstanding that equation 2.9 does not include the effects of diffraction and finite mechanical impedance).

The proper analysis of the Delta-Z method is in terms of a reciprocal two-port network. Such an analysis is given in this section and leads to a different equation for the open-circuit sensitivity than equation 2.9.

A reversible electroacoustic transducer may be represented as a reciprocal two-port network, as schown in figure 3:

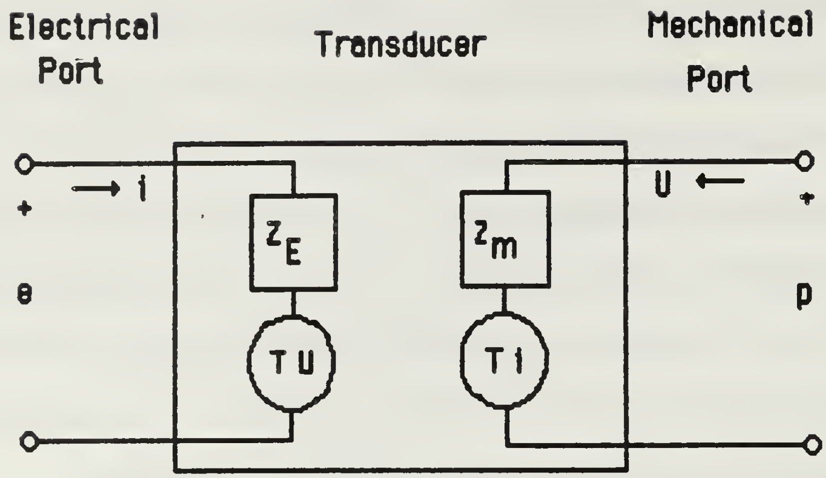

Figure 3. Two-Port Representation of an Electro Acoustic Transducer

The appropriate two-port equations are

$$e = Z_E\, i + T\, U, \tag{2.11}$$

$$p = T\, i + Z_m\, U, \tag{2.12}$$

here:

e = the voltage across the electrical terminals,

i = the current passing through the electrical terminals,

U = the volume velocity directed into the face of the transducer,

p = the average acoustic pressure at the face of the transducer,

Z_E = the electrical impedance with blocked mechanical terminals,

Z_m = the mechanical impedance with blocked (open-circuit) electrical terminals,and

T= the transduction coefficient for a reciprocal transducer.

Now consider such a transducer radiating into air as schematized in Figure 4.

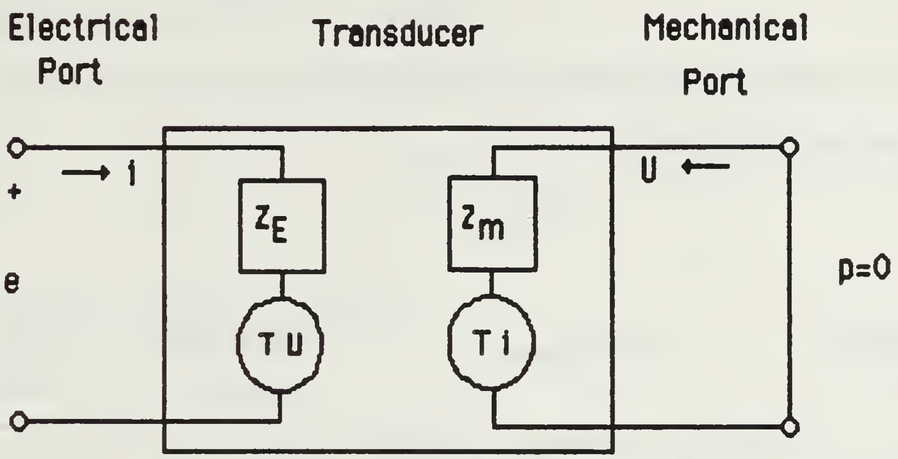

Figure 4. Transducer Radiating in Air

The radiation impedance of the transducer in air is considered to be negligible compared to Z_m and hence is represented in Figure 4 by a short circuit,

$$p = 0. \tag{2.13}$$

Solving for U in equation 2.12 with p=0 and substituting in 2.11:

$$U = \frac{-T\,i}{Z_m},$$

$$e = Z_E\, i + T\left(\frac{-T\,i}{Z_m}\right).$$

Thus the electrical input impedance in air becomes:

$$Z_{air} = Z_E - \frac{T^2}{Z_m} \quad . \tag{2.14}$$

Now consider the transducer transmitting into a free-field in water, as represented by Figure 5.

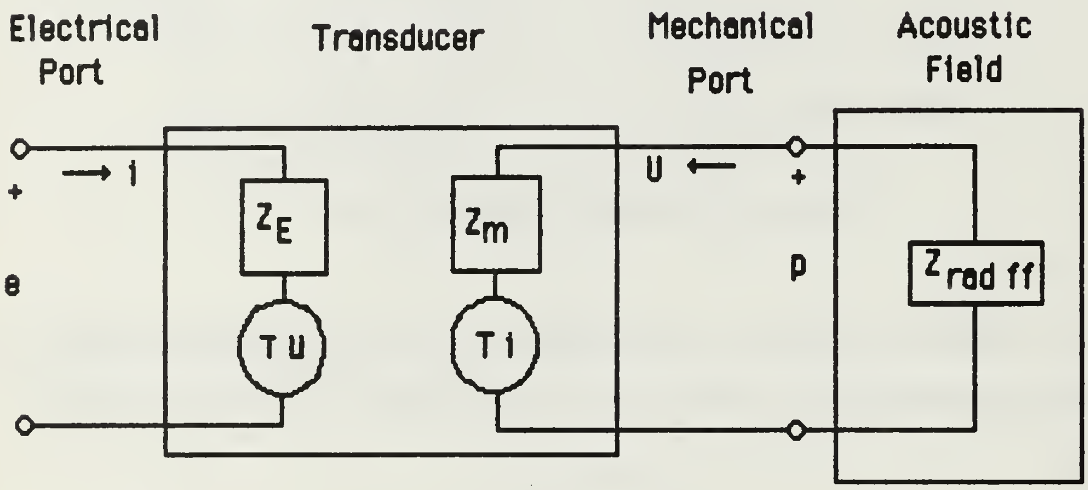

Figure 5. Transducer Radiating in Water

Here $Z_{rad\ ff}$ is the free-field radiation impedance of the transducer in water. Hence:

$$p = -Z_{rad\ ff}\, U \quad . \tag{2.15}$$

Substituting this into equation 2.12 gives

$$-Z_{rad\ ff}\, U = T\, i + Z_m\, U$$

or

$$U = \frac{-T\,i}{(Z_m + Z_{rad\,ff})}\ .$$

Substituting this into equation 2.11 yields

$$e = Z_E\,i + T\left[\frac{-T\,i}{Z_m + Z_{rad\,ff}}\right].$$

The input impedance therefore becomes:

$$Z_{water} = \frac{e}{i} = Z_E - \left[\frac{T^2}{Z_m + Z_{rad\,ff}}\right] \qquad (2.16)$$

and the impedance difference between water and air is given by:

$$\Delta Z = Z_{water} - Z_{air} = Z_E - \frac{T^2}{Z_m + Z_{rad\,ff}} - Z_E + \frac{T^2}{Z_m}$$

which simplifies to

$$\Delta Z = \frac{T^2 Z_{rad\,ff}}{(Z_m + Z_{rad\,ff})Z_m} \qquad (2.17)$$

Thus the electromechanical coupling coefficient T is proportional to the square root of the difference in the input electrical impedance of the transducer when loaded by water and air.

Note that if $Z_{rad\ ff}$ and Z_m are known, then T may be found from ΔZ using equation 2.17. If Z_E is also known, then all the two-port parameters would be known and in principle one may analyze any proposed transducer application. In many cases of interest, however, Z_m and Z_E need not be known to obtain the sensitivity of the transducer, M_0, provided that $|\Delta Z|/|Z|$ is significant. As will be shown below, this simplification is obtained when the radiation impedance is small compared to the mechanical impedance, which is very often the case for piezoelectric ceramic transducers except near frequencies of mechanical resonance. This is the great utility of the Delta-Z method.

To relate the free field open circuit voltage sensitivity, M_0, to ΔZ, imagine an acoustic wave impinging on the transducer. The network representing this situation is depicted in Figure 6.

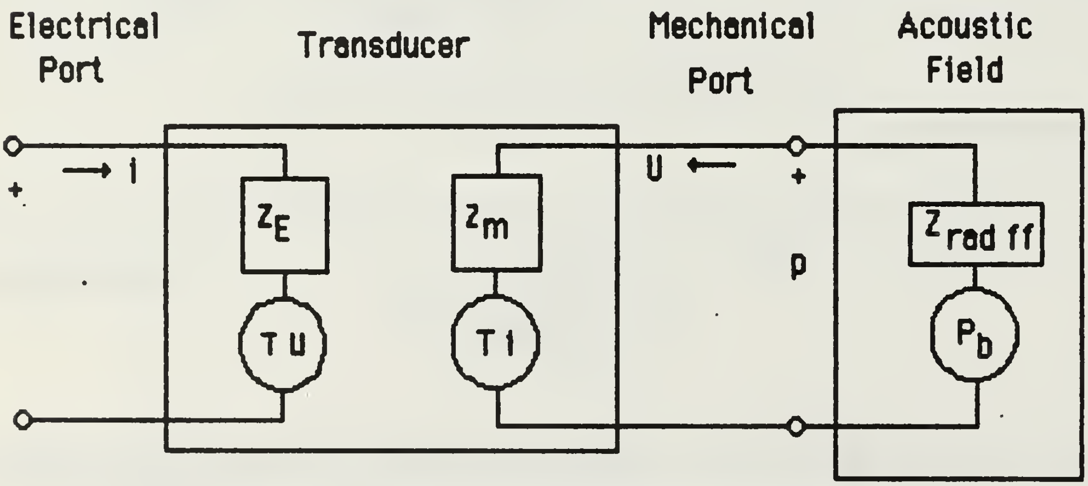

Figure 6. Transducer in an Acoustic Field

Where:

p_b = the blocked pressure, that is, the average pressure over the face of the transducer with zero volume velocity,

p_{ff} = the free-field pressure, that is, the pressure which would exist at the location of the transducer if it were absent,

D = the diffraction constant, and

$Z_{rad\,ff}$ = the free-field radiation impedance of the transducer in water.

By definition, the blocked pressure and the free-field pressure are related by the diffraction constant, D, which depends only on the geometry of the transducer and on the acoustic wavelength:

$$p_b = D\,p_{ff}\,. \tag{2.18}$$

The free-field open circuit (i=0) voltage sensitivity is derived from Figure 6 as follows.

The free-field open-circuit voltage sensitivity may be cast as

$$M_{oc\,ff} = M_o = \frac{e_{oc}}{p_{ff}} = \frac{e_{oc}}{p}\;\frac{p}{p_b}\;\frac{p_b}{p_{ff}}\,. \tag{2.19}$$

When i=0 is substituted in equations 2.11 and 2.12, they become:

$$e_{oc} = T\,U$$

$$p = U\,Z_m$$

giving:

$$\frac{e_{oc}}{p} = \frac{T}{Z_m} \quad . \tag{2.20}$$

Also, from Figure 6,

$$p = p_b - Z_{radff}\,U \tag{2.21}$$

Therefore:

$$p\left[\frac{1 + Z_{rad\,ff}}{Z_m}\right] = p_b = \frac{Z_m + Z_{rad\,ff}}{Z_m}$$

giving:

$$\frac{p}{p_b} = \frac{Z_m}{Z_m + Z_{rad\,ff}} \quad . \tag{2.22}$$

Substituting equations 2.18, 2.20 and 2.22 into 2.19, the open circuit free-field sensitivity becomes:

$$M_o = \frac{T}{Z_m}\left[\frac{Z_m}{Z_m + Z_{rad\,ff}}\right] D \quad . \tag{2.23}$$

The transduction coefficient T is found from equation 2.17:

$$T = \left[\Delta Z\, Z_m \frac{(Z_m + Z_{rad\,ff})}{Z_{rad\,ff}}\right]^{1/2} \quad .$$

Equation 2.23 then becomes:

$$M_o = \left[\Delta Z\, Z_m \frac{(Z_m + Z_{rad\,ff})}{Z_{rad\,ff}}\right]^{1/2} \frac{D}{Z_m} \frac{Z_m}{(Z_m + Z_{rad\,ff})}$$

which simplifies to

$$M_0 = D \left[\frac{\Delta Z\, Z_m}{Z_{rad\ ff}\ (Z_m + Z_{rad\ ff})} \right]^{1/2} \qquad (2.24)$$

which is the desired result.

If $Z_{rad\ ff} \ll Z_m$, as is usually the case for a piezoelectric transducer except near frequencies of mechanical resonance, equation 2.24 becomes much simpler:

$$M_0 = D \left[\frac{\Delta Z}{Z_{rad\ ff}} \right]^{1/2} \qquad (2.25)$$

Note that in this case the mechanical impedance need not be known to determine M_0. This feature makes the Delta-Z method potentially very useful for *in-situ* calibration of in-service transducers. Also note the similarity and difference between this and the Australian result for the sensitivity which was given in section II.B.3 and is reproduced here:

$$M_H = (\Delta Z\, J_p)^{1/2}$$

It is important to note that $Z_{rad\ ff}$ in equation 2.25 is <u>not</u> a reciprocity parameter and should not be construed as the inverse of the planar reciprocity parameter, J_p, even if equation 2.9 took diffraction into account, which it does not. These differences will be apparent in Chapter V where the experimental results of both methods will be compared to the standard comparison calibration of a spherical transducer performed at TRANSDEC, San Diego.

A similar two-port analysis may be performed which relates the short-circuit free-field current sensitivity of a reversible transducer to the difference in the input electrical admittance when the transducer is loaded by air and water. The result is

$$M_{sc} = D\left[\frac{\Delta Y \; Y_{rad\,ff}}{(1+Y_m Z_{rad\,ff})}\right]^{1/2}, \qquad (2.26)$$

where:

M_{sc} = the short-circuit free-field sensitivity,

ΔY = the change of admittance between the water and air,

$Y_{rad\,ff}$ = the radiation admittance of the transducer,

Y_m = the mechanical admittance with shorted electrical terminals, and

Y_E = the electrical admittance with shorted mechanical terminals.

This formula may be more appropriate for a low mechanical impedance device such as a moving coil transducer. (Note that a moving coil transducer is anti-reciprocal and that we must replace T^2 in the above by $-T^2$. This has no effect on the magnitude of the final result.).

In the case where

$$Y_m Z_{rad\,ff} << 1,$$

then equation 2.26 simplifies to

$$M_{sc} = D\,(\Delta Y \; Y_{rad\,ff})^{1/2} \;. \qquad (2.27)$$

The open-circuit voltage sensitivity can be determined using admittance parameters instead of impedance parameters with the following result:

$$M_0 = \frac{-T\,D}{Y_E + (Y_E\,Y_m - T^2)Z_{rad\,ff}} \qquad (2.28)$$

where:

T = the transduction coefficient for a reciprocal transducer, and

$T = [\,\Delta Y\,(Y_m + Y_{rad\,ff})]^{1/2}$.

III. DESCRIPTION OF EXPERIMENT AND RAW DATA

A. DESCRIPTION OF THE TRANSDUCER

The transducer used for this experiment was a 6-inch outer diameter, hollow, piezoelectric sphere. It was borrowed from TRANSDEC and was built at NOSC, San Diego. It consists of two radially poled piezoelectric ceramic hemi-spheres of unknown (probably about 3/8") thickness glued together. It is potted in polyurethane of approximately 1/4 inch thickness and has a 12 inch long hard rubber handle to house the connection to the 100 foot long electrical cable (Cable # 71102, DSS-3 (RF), MIL-C-915A), as depicted in Figure 7. Its low frequency capacitance is 210 nF (@1000 Hz).

B. EXPERIMENTAL SET-UP AND PROCEDURES

Measurements of Z_{water} and Z_{air} were carried out at TRANSDEC, NOSC, San Diego. The main feature of this facility is a large water tank which simulates free-field conditions for a transducer placed at a certain position (6m depth at the center of the pool) [Ref. 6]. Its unique construction focuses the sound energy on the outside perimeter of the tank where it is reflected into a sound trap, eliminating the energy reflected back to the transducer.

Measurements were conducted using a continuous wave (CW) signal generated from the HP-4192 Impedance Analyzer which was controlled by a HP-9826 computer and HP Basic 4.0 software. The equipment set-up of Figure 7 was used to determine the resistance (R) and reactance (X) components of the impedance (Z) of the spherical transducer in both water and air.

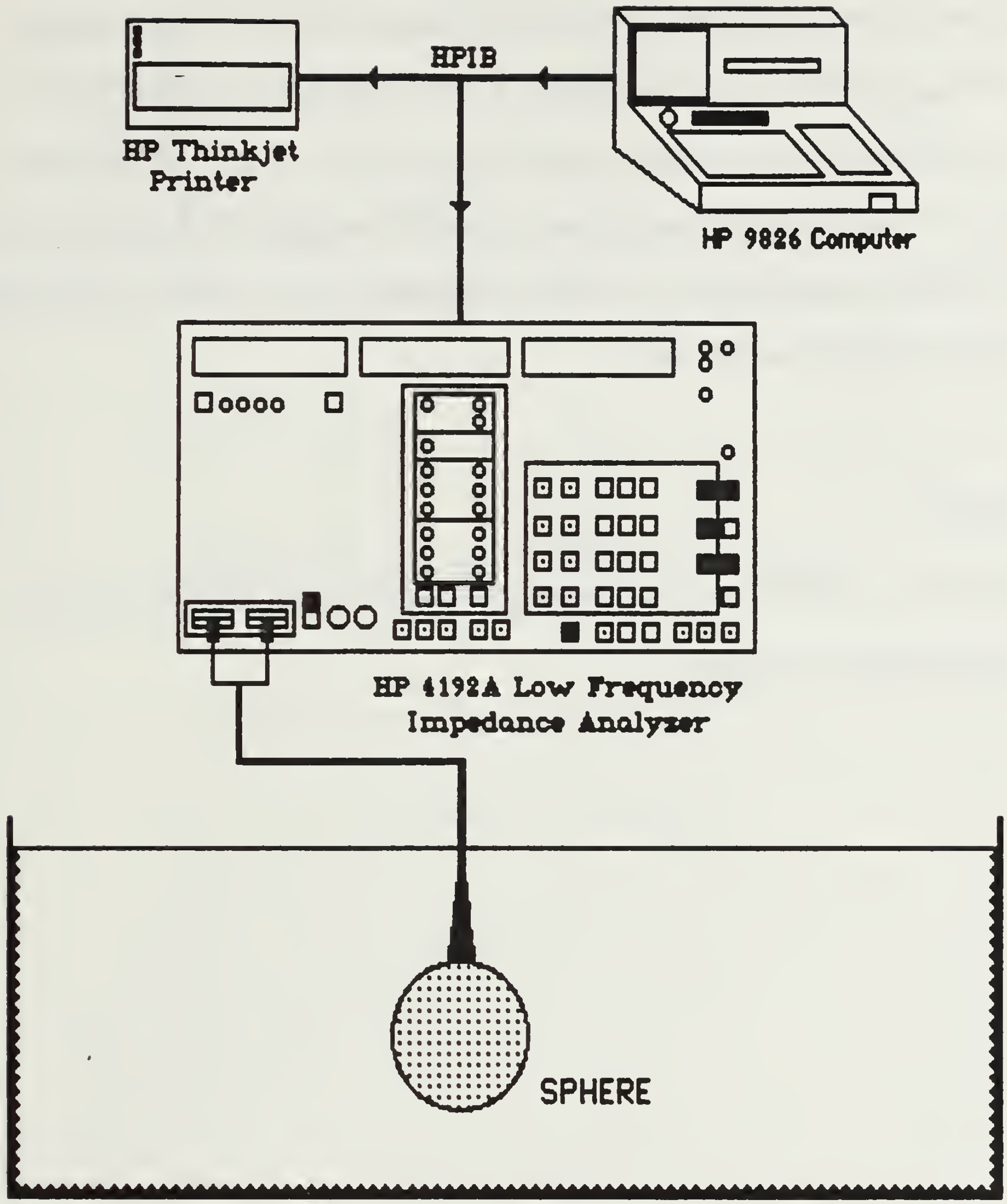

Figure 7 : Equipment Configuration Used to Determine Sensitivity by the Delta-Z Method .

The procedure was as follows:

The sphere was placed 6 m below the surface of the water for half an hour to allow its temperature to reach equilibrium with the surrounding water. The R_{water} and X_{water} components were measured and stored on a floppy disk. The data covered the range of 1.0 kHz to 15.0 kHz in 100 equal increments. The sphere was then pulled out of the water, and R_{air} and X_{air} were immediately measured at the same frequencies as in water and stored on the same file as the water data.

C. RAW DATA

Figures 8 to 11 show the raw impedance data obtained for the sphere following the above procedure.

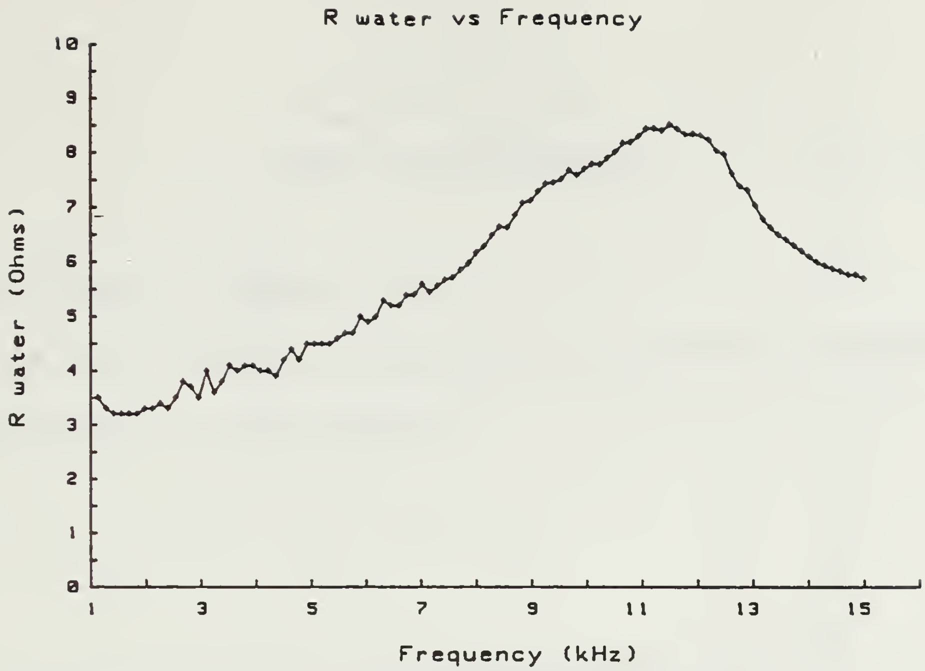

Figure 8. R_{water} as a Function of Frequency.

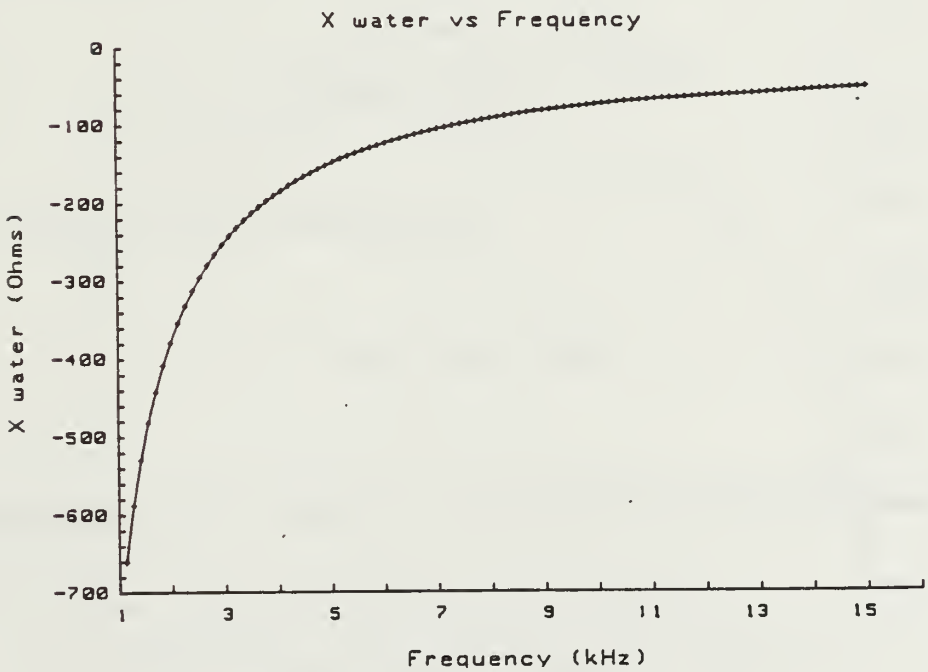

Figure 9 . X_{water} as a Function of Frequency.

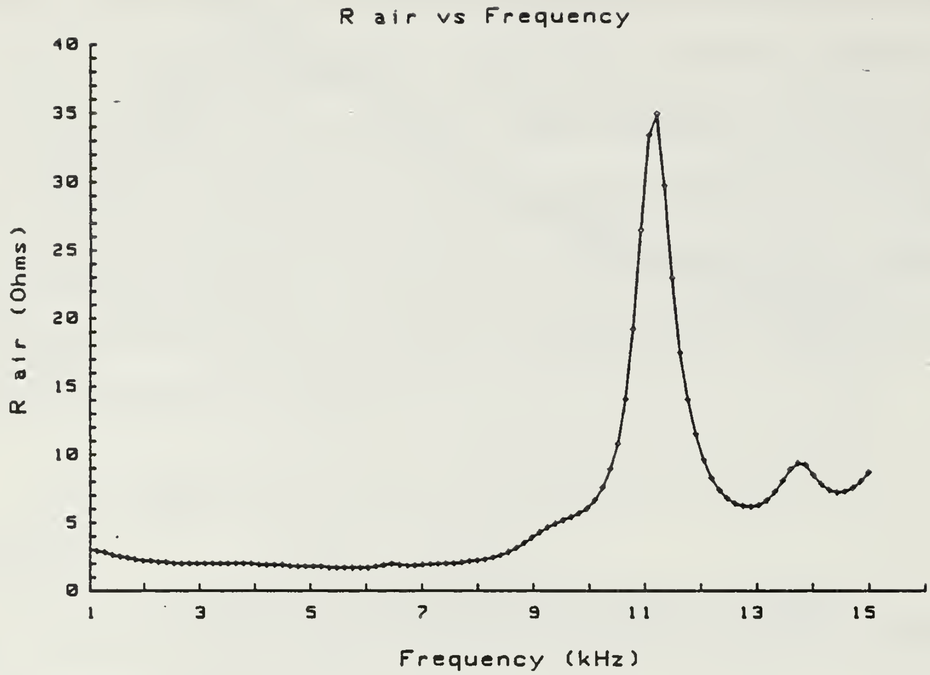

Figure 10. R_{air} as a Function of Frequency.

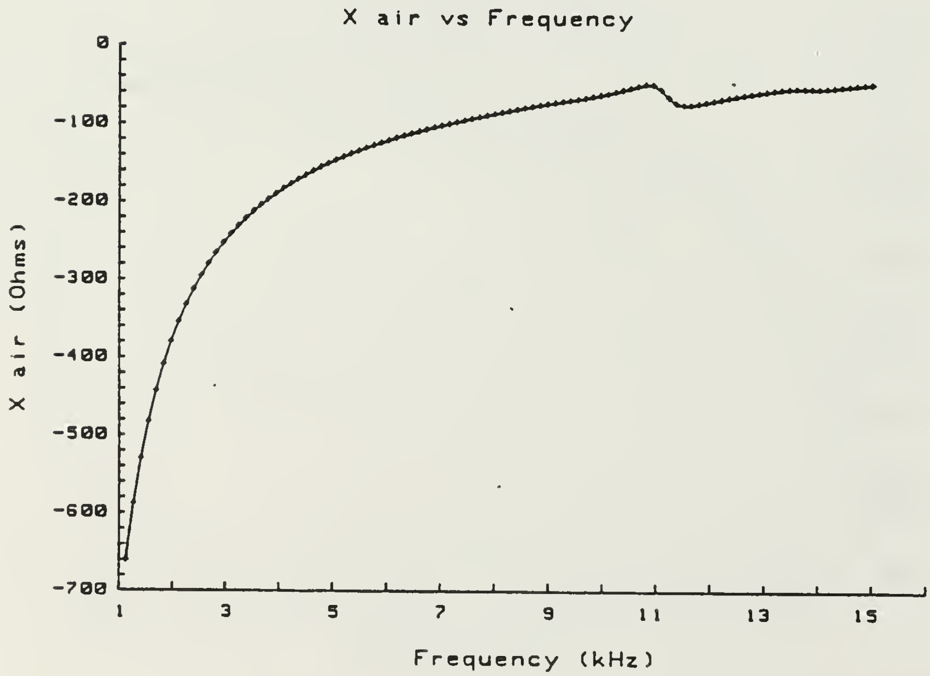

Figure 11. X_{air} as a Function of Frequency.

IV. ANALYSIS AND RESULTS

A. EXTRACTION OF ΔZ FROM RAW DATA

The electrical impedance measured at the terminals of the transducer is represented by the complex quantity:

$$Z = R + jX$$

where :

Z = impedance,

R = resistance, and

X = reactance.

For the measurements taken in water the expression is written:

$$Z_{water} = R_{water} + j X_{water}$$

and similarily for the air measurements:

$$Z_{air} = R_{air} + j X_{air}.$$

The magnitude of ΔZ for insertion in the sensitivity calculations is given by:

$$|\Delta Z| = \left[(R_{water} - R_{air})^2 + (X_{water} - X_{air})^2 \right]^{1/2} \tag{4.1}$$

and the angle associated with it is:

$$\angle \Delta Z = \arctan \left[\frac{X_{water} - X_{air}}{R_{water} - R_{air}} \right] \tag{4.2}$$

Only the magnitude of ΔZ was used in this experiment.

The values of resistance and reactance used in equation 4.1 were obtained as described in section III.B. Figure 12 shows a plot of $|\Delta Z|/|Z_{water}|$ vs frequency.

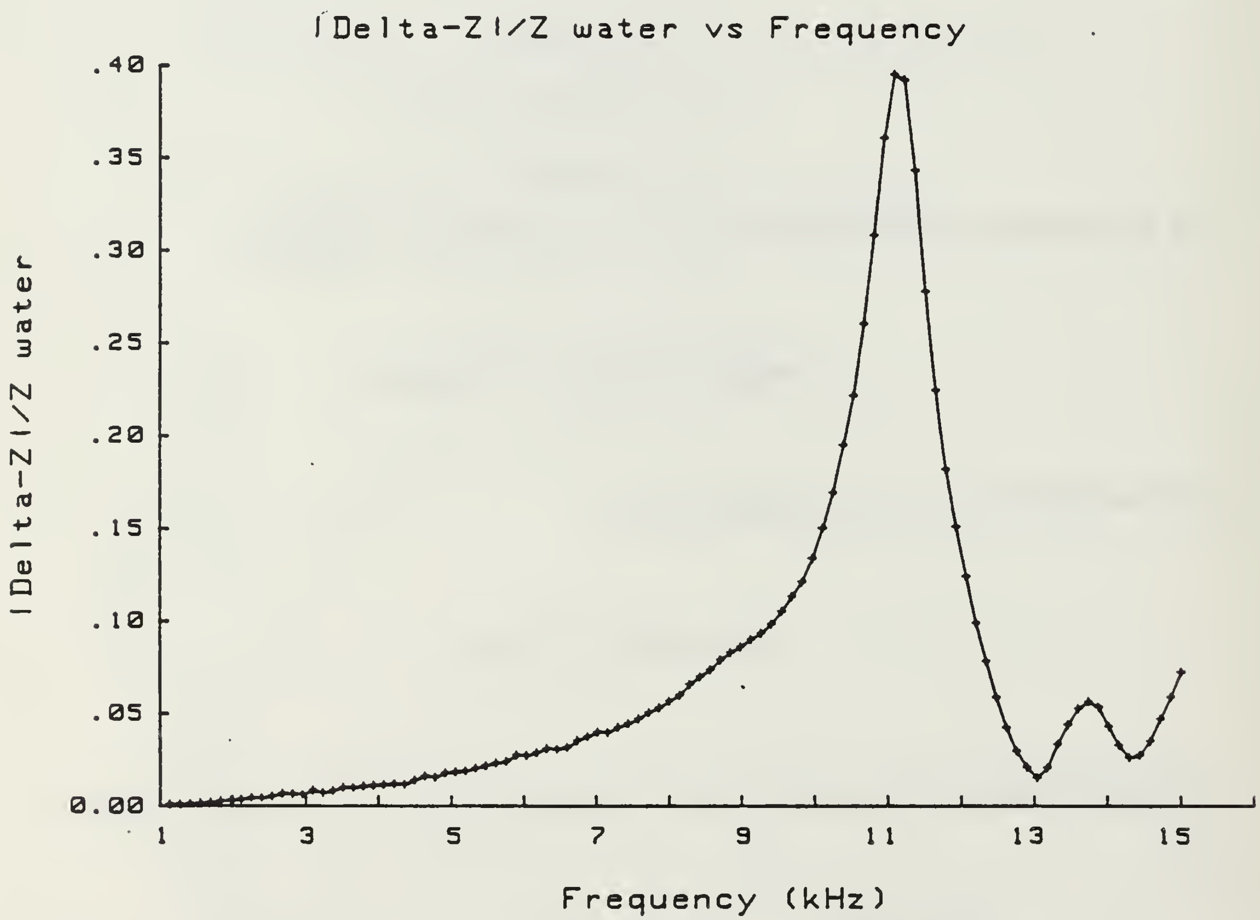

Figure 12. Plot of $|\Delta Z|/|Z_{water}|$ vs Frequency

B. VARIABILITY OF ΔZ MEASUREMENTS

To ascertain the reliability of the values of ΔZ, the experiment was repeated three different times. Figure 13 depicts the results obtained for $|\Delta Z|/|Z_{water}|$ from these three independent experiments.

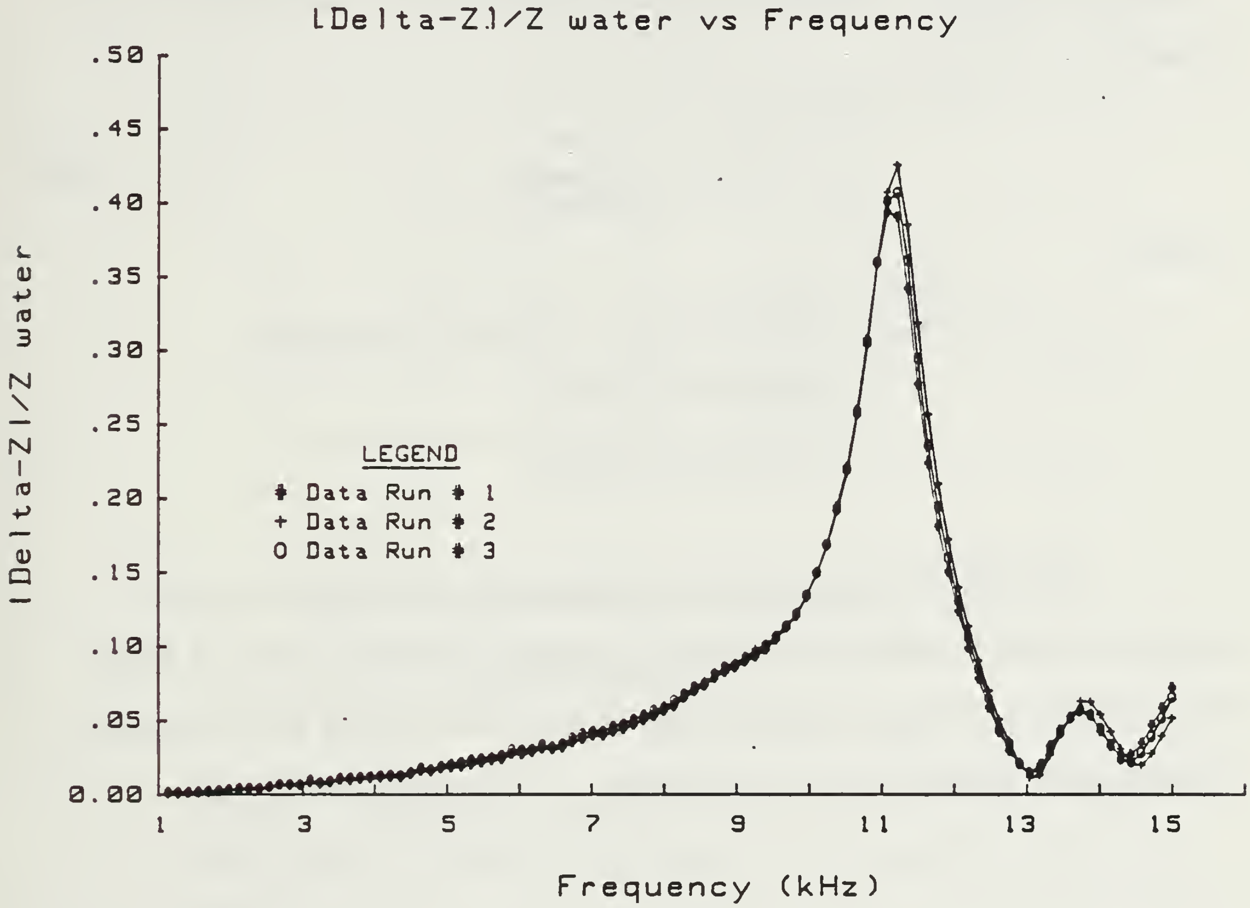

Figure 13. Plot of $|\Delta Z|/|Z_{water}|$ for Three Independent Measurements

The results obtained indicate that random experimental error is probably less than 1% for frequencies below resonance. A detailed error analysis is included in Appendix A. Determination of the remaining parameters needed to calculate Mo (D, $Z_{rad\ ff}$, Z_m) is described below.

C. DETERMINATION OF D, $Z_{rad\,ff}$ and Z_m

1. Diffraction constant (D)

Recall from Figure 6 and the discussion which followed that the diffraction constant is defined to be the ratio of the blocked pressure to the free-field pressure. It was derived by Henriquez for a sphere [Ref. 7] and is given by :

$$D = \frac{1}{\sqrt{1 + (kr)^2}} \tag{4.3}$$

where:

D = magnitude of the diffraction constant,

k = wave number = ω/c,

a = radius of the sphere.

The diffraction constant of a transducer with a more complex geometry is not necessarily a simple expression and may have to be found experimentally. Figure 14 depicts the diffraction constant as a function of frequency for several interesting shapes.

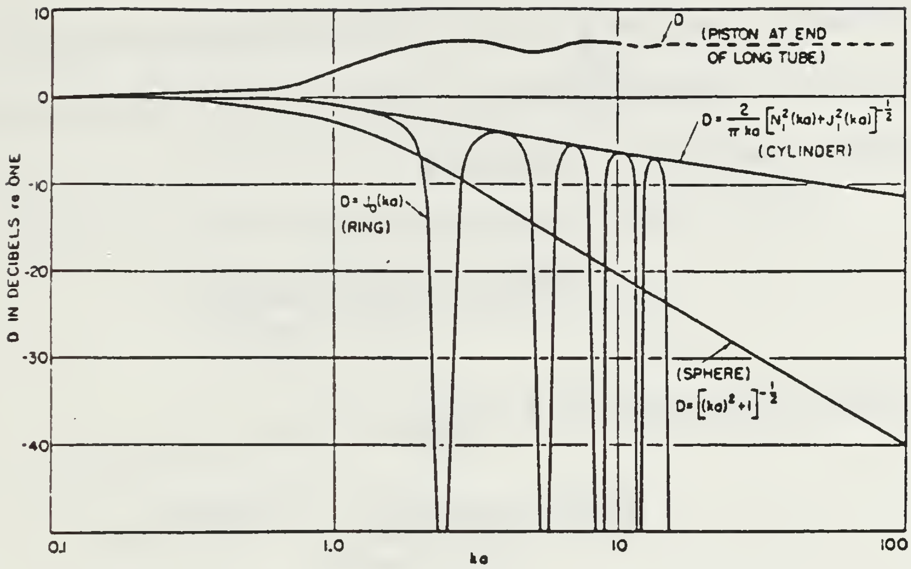

Figure 14. Diffraction Constant as a Function of Frequency [Ref. 7]

2. Radiation Impedance ($Z_{rad\ ff}$)

The radiation impedance of a sphere in radial motion is given by Beranek [Ref. 8]:

$$Z_{rad\ ff} = \text{Radiation impedance in free-field}$$
$$= \frac{j\omega\rho_0 ca}{S\left(c + j\omega a\left(1 - \frac{jc\delta}{\omega}\right)\right)} \tag{4.4}$$

where:

ω = radial frequency,

ρ_0 = density of the medium,

c = speed of sound propagation in the medium,

S = surface area of the sphere,

a = radius of the spherical transducer, and

δ = absorbtion constant.

Since $c\delta/\omega$ is in the order of 1×10^{-3} for all frequencies of interest in this study, it can be neglected without significantly affecting the results [Ref. 8, pp 57,58]. Therefore it is assumed:

$$Z_{rad\ ff} = \frac{j\rho_0 c\,(ka)(1-jka)}{S\left[1+(ka)^2\right]} \qquad (4.5)$$

The real and imaginary parts of equation 4.5 are :

$$R_{rad} = \frac{\rho_0 c(ka)^2}{S(1+(ka)^2)} = (ka)\ X_{rad} \qquad (4.6)$$

$$X_{rad} = \frac{\rho_0 c\,ka}{S(1+(kr)^2} \qquad (4.7)$$

3. Blocked Mechanical Impedance (Z_m)

The blocked mechanical impedance of the sphere was extracted from its input electrical impedance in air and water as described below. The success of the technique employed depends upon the assumption that the radiation resistance in water is significant compared to the blocked mechanical resistance.

The input electrical impedance of a simple reversible piezoelectric ceramic transducer in air may be modeled by the approximate equivalent circuit shown in Figure 15 [Ref. 9].

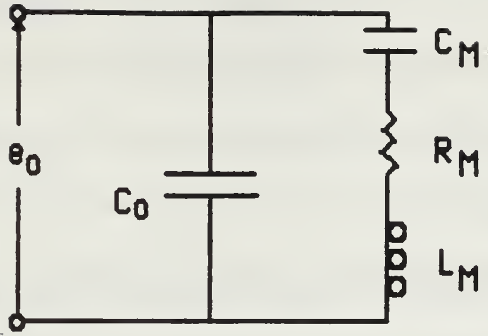

Figure 15. Approximate Equivalent Circuit of a Simple Piezoelectric Transducer in Air.

where:

e_0 = open circuit voltage,

C_0 = blocked electrical capacitance,

C_M = blocked mechanical capacitance,

L_M = blocked mechanical inductance, and

R_M = blocked mechanical resistance.

The notation follows Wilson [Ref. 9], i.e. capital letter subscripts refer to equivalent electrical quantities, lower case subscripts refer to mechanical quantities. Only one mechanical resonance has been modeled in Figure 15; in principle any number of additional resonances can be modeled by adding one more parallel R-L-C branch for each.

In making the above electrical analogy it is assumed the vibrating elements are mechanically equivalent to a single degree-of-freedom mass-spring-dashpot system and that the radiation impedance in air can be neglected. It is also assumed that the equivalent circuit parameters are not

frequency dependent. The latter assumption is not crucial to the success of the Delta-Z method, however, since the value of the mechanical impedance significantly affects the calculated sensitivity only near frequencies of mechanical resonance.

When the transducer is surrounded by water the radiation impedance cannot be neglected and is represented in Figure 16 by R_{RAD} and L_{RAD}.

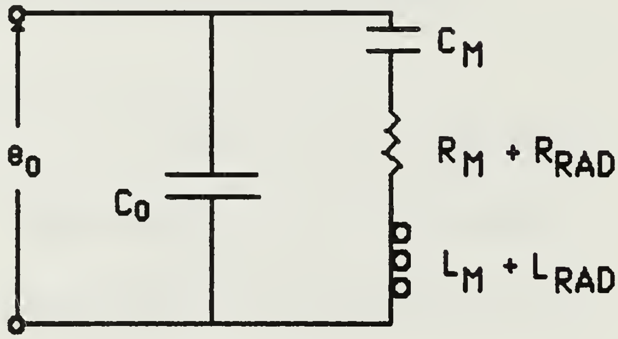

Figure 16. Approximate Equivalent Circuit of a Simple Piezoelectric Transducer in Water.

The procedure used to extract the mechanical impedance of the sphere, or, more precisely, the mechanical resistance R_m, inertance L_m, and compliance C_m, is as follows.

At the frequency of mechanical resonance in air (ω_0), the effects of L_M and C_M cancel and leave R_M in parallel with C_0 (see Figure 15), giving the following expression for the input electrical impedance:

$$Z_{air} = \frac{R_M(1-j\omega_0 R_M C_0)}{1+(\omega_0 R_M C_0)^2} \quad (4.8)$$

where :

ω_0 = the natural mechanical resonance frequency.

Separating Z_{air} into its real and imaginary parts we obtain:

$$R_{air} = \frac{R_M}{1+(\omega_0 R_M C_0)^2} \quad (4.9)$$

and

$$X_{air} = \frac{-R_M^2 \omega_0 C_0}{1+(\omega_0 R_M C_0)^2} \quad (4.10)$$

Rearranging equations 4.9 and 4.10 gives:

$$\frac{X_{air}}{R_{air}} = -\omega_0 R_M C_0 . \quad (4.11)$$

$$R_{air}^2 + X_{air}^2 = \frac{R_M^2}{1+(\omega_0 R_M C_0)^2} \quad (4.12)$$

Values of R_{air} and X_{air} values are obtained from the input electrical impedance data with the transducer in air. R_M and C_0 are then easily found by inverting equations 4.11 and 4.12. The same procedure is used to find the values of ($R_M + R_{RAD}$) and C_0 with the transducer in water (Figure 16). The values of R_M, C_0, and R_{RAD} at resonance found this way are:

$$R_M = 129\ \Omega$$

$$C_0 = 209.\text{nF}$$

$$R_{RAD} = 1.89 \times 10^7\ \Omega.$$

Using the fact that the transformation of the mechanical resistances, R_m and R_{rad}, into their equivalent electrical quantities, R_M and R_{RAD}, is identical, the following may be written:

$$R_m = \frac{R_M \; R_{rad}}{R_{RAD}} \qquad (4.13)$$

Hence R_m may be found from this equation using the values of R_{RAD} and R_M extracted previously once R_{rad} is known. Since R_{rad} can be easily calculated for the sphere, this is a simple procedure. The determination of R_{rad} will be one of the crucial factors on which the successful application of the Delta-Z method to *in-situ* transducer calibration will depend. (It is not necessary to know R_{rad}, however to monitor degradation in sensitivity.)

It should be noted that the ratio R_m/R_{rad} is related to the electroacoustic efficiency of a piezoelectric transducer, η, as defined by Wilson [Ref. 9,p. 39] and Hunt [Ref. 10], by:

$$\frac{R_M}{R_{RAD}} = \frac{R_m}{R_{rad}} = \frac{1-\eta}{\eta} \qquad (4.14)$$

so that equation 4.13 may be written

$$R_m = \frac{1-\eta}{\eta} R_{rad}$$

Having found R_m from equation 4.13, L_m and C_m are found from the center frequency, ω_0, and quality factor, Q, of the mechanical resonance using

$$L_m = \frac{Q \, R_m}{\omega_0} \qquad (4.15)$$

and

$$C_m = \frac{1}{\omega_0 Q R_m} . \tag{4.16}$$

The resulting values of R_m, L_m and C_m for the sphere are:

$$R_m = 5.93 \times 10^{6} \text{ N sec/m}^5$$

$$L_m = 1.25 \times 10^{3} \text{ N sec/m}^5$$

$$C_m = 1.46 \times 10^{-13} \text{ m}^5\text{/N sec},$$

and the electrical values of the electrical equivalent values R_M, L_M and C_M

are

$$R_M = 129 \ \Omega$$

$$L_M = 27.2 \text{ mH}$$

$$C_M = 6.7 \text{ nF},$$

Figure 17 shows a plot of Z_m calculated using these values to the radiation impedance calculated for the sphere.

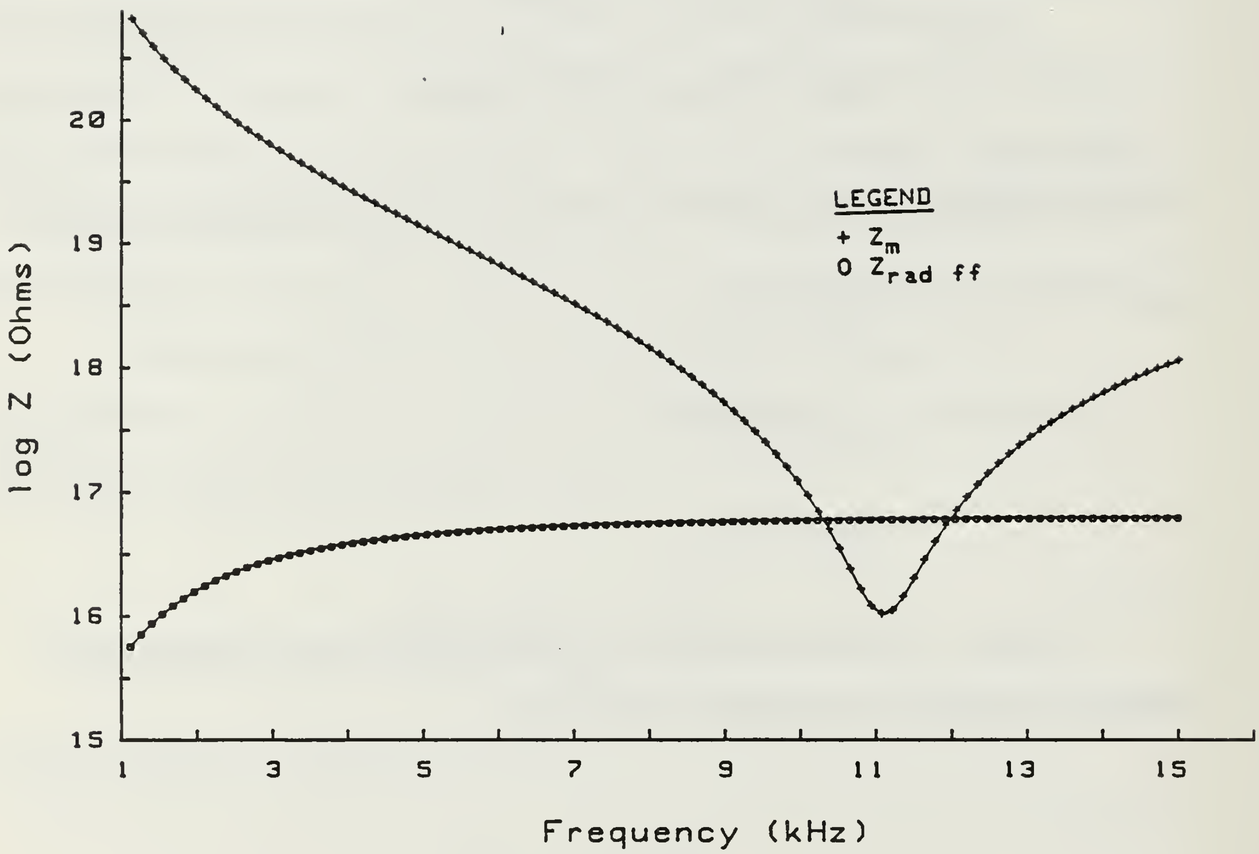

Figure 17. Z_m and $Z_{rad\ ff}$ as a Function of Frequency

V. DISCUSSION

A. COMPARISON OF DELTA-Z METHOD TO STANDARD CALIBRATION METHOD

The standard of comparison for all other calibration methods used to determine the sensitivity of the spherical transducer is the comparison calibration described in Section II.A.1. The comparison calibration of the spherical transducer was obtained at the Transducer Evaluation Center (TRANSDEC) at the Naval Ocean Systems Center (NOSC), San Diego, Ca, using a Navy standard type F-37 transducer. The comparison calibration method is widely used by the Navy and most other organizations that require absolute calibration of hydrophones.

Figure 18 shows the results obtained at TRANSDEC by the Delta-Z method using eq. 2.24 (curve labeled "Delta-Z Method") compared to those obtained from the standard comparison calibration method. The two methods give results which agree within several dB except at the extremes of the frequency range of the experiment. The results from the Delta-Z method should be disregarded for frequencies above approximately 12 kHz since the second and third mechanical resonances occur in this region and were not accounted for in the estimation of Z_m. Also, the results from the comparison calibration are suspect at frequencies below several kHz, as the pool at TRANSDEC cannot be considered anechoic at these low frequencies. In the frequency range 3 to 12 kHz the largest discrepancy between the results of the two methods is approximately 3 dB while the results over most of the frequency range agree within about 1 dB.

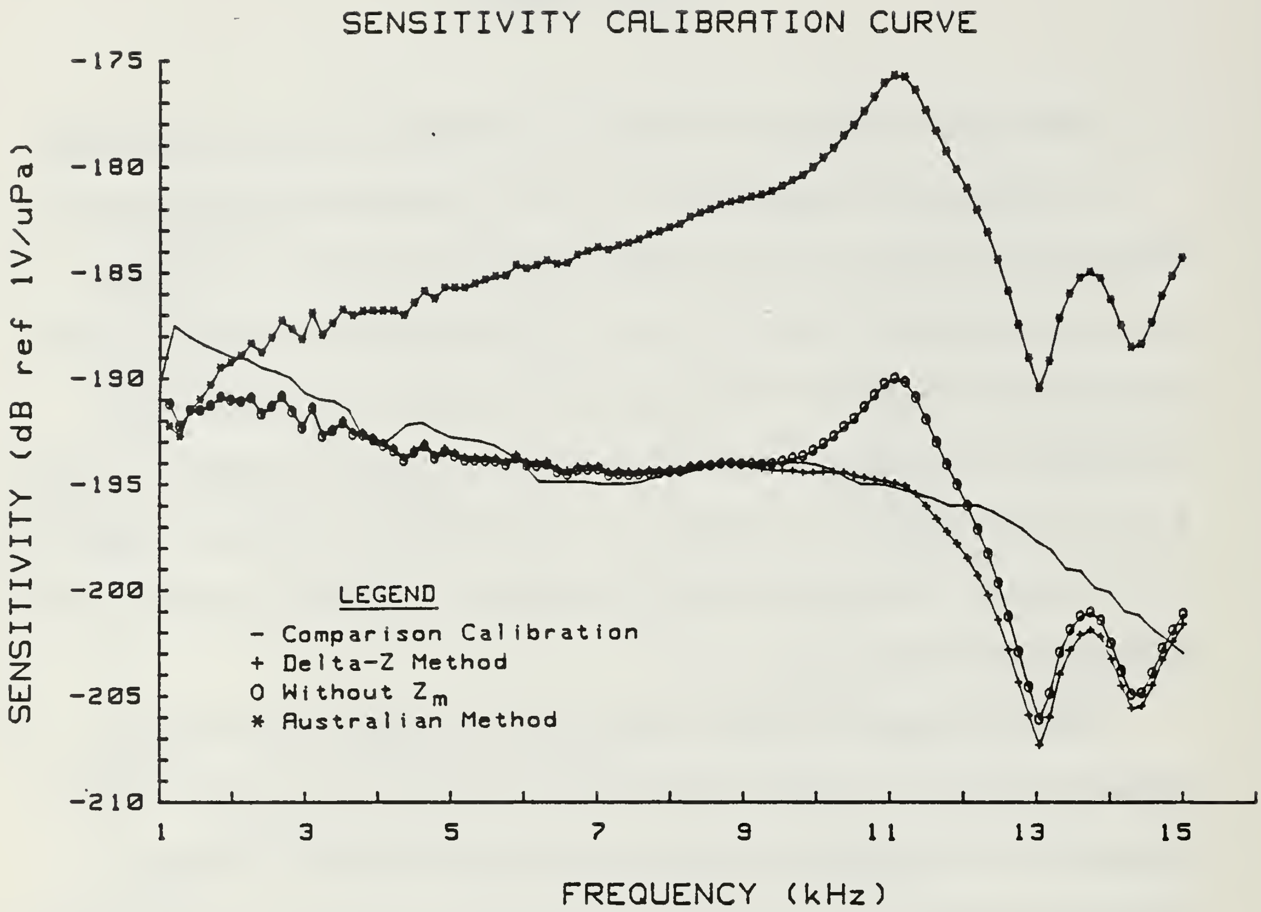

Figure 18. Sensitivity Obtained From Various Calibration Methods

The importance of including the effects of finite mechanical impedance in the Delta-Z method in order to obtain accurate results near frequencies of mechanical resonance is readily apparent. The curve labeled "Without Z_m" in Figure 18 was calculated using equation 2.25, which assumes $Z_m >> Z_{rad\ ff}$. The error made at 11 kHz by not accounting for finite Z_m amounts to more than 5 dB.

The close agreement between the results of the Delta-Z and comparison calibration methods below the frequencies of mechanical resonance was quite unexpected and is very encouraging for the prospect of applying this method to the monitoring of in-service transducers. Before the Delta-Z experiment was performed it was anticipated that the precision of the results would be best for frequencies around mechanical resonance, as Z_m is minimum there and the effect of water loading would therefore be more observable, i.e. $|\Delta Z|/|Z|$ would be larger. It was for this reason that a relatively large (by laboratory standards: 6" diameter) sphere was chosen for the experiment. Based on the experimental results there is good reason to believe that if the diffraction constant and the free-field radiation impedance can be determined within a few percent, the sensitivity of an in-service transducer can be determined within 1 dB if ΔZ can be determined within 2 dB. From the magnitude of $\Delta Z/Z$ observed in this experiment (typically a few percent, see Figure 12), the precision with which impedance needs to be measured is probably about 0.1%.

B. COMPARISON OF THE ROYAL AUSTRALIAN NAVY METHOD TO STANDARD CALIBRATION METHOD

As was described in section II.B.3, the equation resulting from the Royal Australian Navy analysis of the Delta-Z method is:

$$M_H = (\Delta Z \, J_p)^{1/2}$$

where:

M_H = open-circuit sensitivity of the transducer,

ΔZ = the change of input electrical impedance between water and air, and

J_p = the planar reciprocity parameter.

The results obtained using this equation are labeled "Australian Method" in Figure 18. The sensitivity calculated using the RAN method is in great disagreement with that calculated using the other two methods.

VI. SUMMARY, CONCLUSION AND RECOMMENDATIONS

A. INTRODUCTION

The objectives of the research described in this thesis were 1) to develop an underwater transducer calibration procedure based on the difference in input electrical impedance when loaded by water and air, and 2) to experimentally test the procedure using a simple geometry (spherical) transducer.

B. SUMMARY

Using reciprocal two-port network theory, an equation for the free-field open-circuit voltage sensitivity of a reversible underwater acoustic transducer was derived in terms of the difference in input electrical impedance when loaded by water and air. The theory takes into account the diffraction due to the finite size of the transducer, its finite mechanical impedance, and its free-field radiation impedance in water. An experiment to test the predictions of the theory was performed using a 6" diameter, hollow, piezoelectric ceramic spherical transducer. The results of the experiment agreed within several dB with a standard comparison calibration over the frequency range for which both the Delta-Z analysis and the comparison calibration data are considered reliable.

C. CONCLUSIONS

It has been established that the open-circuit free-field voltage sensitivity of a reversible underwater piezoelectric acoustic transducer can be obtained by the so-called Delta-Z method, that is, from the difference in its input electrical impedance when loaded by water and air. The appropriate formula, derived from reciprocal two-port network analysis, is given by

$$Mo = D \left[\frac{\Delta Z \, Z_m}{Z_{rad\,ff} \, (Z_m + Z_{rad\,ff})} \right]^{1/2} .$$

The finite mechanical impedance of such a transducer must be taken into account in order to obtain precise results near frequencies of mechanical resonance. Apart from such frequency regions the radiation impedance is generally negligible compared to the mechanical impedance and the sensitivity can be found from the approximate formula:

$$Mo = D \left[\frac{\Delta Z}{Z_{rad\,ff}} \right]^{1/2} .$$

In this case neither the blocked impedance nor the open-circuit mechanical impedance need to be known to determine the open-circuit sensitivity.

The Delta-Z method should not be regarded as a high precision method -- the precision of the results rests on the precision with which the difference in two quantities which often are nearly equal can be measured. Nevertheless, because it is such a simple procedure, it may be useful for in-situ monitoring of in-service transducers in installations which can be flooded and purged.

D. RECOMMENDATIONS

Recommendations for further study are:

1. Investigate the Delta-Z calibration method using piezoelectric ceramic transducers of more complex geometry with the aim of applying this method to the *in-situ* monitoring of in-service transducers.
2. Investigate the application of the Delta-Z method to a low mechanical impedance transducer such as moving coil transducer

APPENDIX

ERROR ANALYSIS

A. GENERAL

Three types of errors are considered in this analysis:

1. Random error, which is an indication of the repeatability of a measurement. This was determined experimentally by repeating the ΔZ measurement three times as described in Section IV.B.
2. Estimated calibration error, which is an approximation of the systematic error of each measurement.
3. Approximation error, values for the different parameters used in the calculation of sensitivity had to be truncated and/or estimated to a certain precision.

B. ERROR IN ΔZ

$$|\Delta Z| = \left[(R_{water}-R_{air})^2 + (X_{water}-X_{air})^2\right]^{1/2}$$

where:

ΔZ = change of impedance for the transducer from water into air,

R_{water} = Resistance of the transducer in water,

R_{air} = Resistance of the transducer in air,

X_{water} = Reactance of the transducer in water,

X_{air} = Reactance of the transducer in air.

Using standard statistical analysis, the standard deviation for the ΔZ parameter is:

$$\frac{S_{\Delta Z}}{\Delta Z} = \left[\left(\frac{\delta \Delta Z}{\delta R_{water}}\right)^2 (S_{R_{water}})^2 + \left(\frac{\delta \Delta Z}{\delta R_{air}}\right)^2 (S_{R_{air}})^2 + \left(\frac{\delta \Delta Z}{\delta X_{water}}\right)^2 (S_{X_{water}})^2 + \left(\frac{\delta \Delta Z}{\delta X_{air}}\right)^2 (S_{X_{air}})^2\right]^{1/2} .$$

The above assumes that R_{water}, R_{air}, X_{water} and X_{air} are statistically independent.

If we further assume that

$S_{R_{water}} = S_{R_{air}} = S_{X_{water}} = S_{X_{air}} = S$, then the fractional deviation of ΔZ is given by

$$\frac{S_{\Delta Z}}{\Delta Z} = \left[\frac{2\,S^2\,(R_{water} - R_{air})^2 + 2\,S^2\,(X_{water} - X_{air})^2}{(R_{water} - R_{air})^2 + (X_{water} - X_{air})^2}\right]^{1/2} ;$$

which leads to

$$\frac{S_{\Delta Z}}{\Delta Z} = \frac{2\ S}{\Delta Z}$$

From the HP-4192A handbook; S = 0.3% of reading, so that

$$\frac{S_{\Delta Z}}{\Delta Z} = 0.6\%$$

C. ERROR IN THE BLOCKED MECHANICAL IMPEDANCE, Z_m

$$Z_m = R_m + j(\omega L_m + 1/\omega C_m) = R_m + j X_m \ .$$

where ω is the radial frequency and L_m and C_m are given in terms of R_m by

$$L_m = \frac{Q R_m}{\omega_0} \quad \text{and} \quad 1/C_m = \omega_0 Q R_m \ .$$

where:

ω_0 = mechanical resonance frequency (rad/sec)

Q = quality factor of mechanical resonance.

Then

$$\delta L_m = \frac{Q \, \delta R_m}{\omega_0} \quad \text{and} \quad \delta(1/C_m) = \omega_0 Q \, \delta R_m \ .$$

Ignoring the errors in f, ω_0 and Q, compared to the error in R_m, then,

$$\delta Z_m = 1 + j\left(\frac{2\pi f Q}{\omega_0} + \frac{\omega_0 Q}{2\pi f}\right) \delta R_m$$

and

$$\frac{S_{Z_m}}{Z_m} = \frac{S_{R_m}}{R_m}$$

Assuming an estimated 10% error on the blocked mechanical resistance R_m (from the estimated 10% error on the equivalent electrical resistance R_M),

$$\frac{S_{Z_m}}{Z_m} \sim 10\%$$

It should be noted that the influence of errors in Z_m on the error in M_0 will only be significant near resonance, where Z_m is of the same order of magnitude as $Z_{rad\ ff}$.

D. ERROR IN THE RADIATION IMPEDANCE, $Z_{rad\ ff}$

For the spherical transducer used in this project:

$$Z_{rad\ ff} = \frac{\rho_0 c\, ka\,(ka + j)}{S\ [1 + (ka)^2]}$$

$$|Z_{rad\ ff}| = \frac{\rho_0 c\, ka}{S\ \left[1 + (ka)^2\right]^{1/2}} = \frac{(\rho_0 c)\ \left[1 + (ka)^{-2}\right]^{-1/2}}{S}$$

taking the log of both sides and differentiating:

$$\frac{\delta |Z_{rad\ ff}|}{|Z_{rad\ ff}|} = \frac{\delta \rho_0 c}{\rho_0 c} - \frac{\delta S}{S} + \frac{\delta(ka)}{(ka)^3}\ \frac{1}{\left[1 + (ka)^{-2}\right]}$$

$$= \frac{\rho_0 c}{\rho_0 c} - \frac{\delta S}{S} + \frac{\delta(ka)}{(ka)}\ \frac{1}{\left[1 + (ka)^2\right]}$$

but $\dfrac{\delta S}{S} = -2\dfrac{\delta a}{a} = -2\dfrac{\delta(ka)}{ka}$, ignoring the error in k

and $\dfrac{1}{\left[1 + (ka)^2\right]} \leqslant 1$

so that $\left| \dfrac{-\delta S}{S} + \dfrac{\delta(ka)}{(ka)} \dfrac{1}{\left[1 + (ka)^2\right]} \right|$

lies between $\dfrac{\delta(ka)}{ka}$ and $2\dfrac{\delta(ka)}{ka}$

then

$$\frac{S_{Z_{rad\ ff}}}{Z_{rad\ ff}} \leqslant \left[\left(\frac{S_{\rho_0 c}}{\rho_0 c}\right)^2 + 2\left(\frac{\delta(ka)}{ka}\right)^2 \right]^{1/2}$$

$\equiv \dfrac{2\,S(ka)}{ka}$, ignoring the error in $\rho_0 c$.

The fractional error in a, therefore in ka, is estimated to be about 5%. hence

$$\frac{S_{|Z_{rad\ ff}|}}{|Z_{rad\ ff}|} \sim 10\%$$

E. ERROR IN THE DIFFRACTION CONSTANT, D

$$D = \frac{1}{\sqrt{1 + (ka)^2}}\ .$$

$$\frac{S_D}{D} = \frac{(ka)^2 S_{(ka)}}{1+(ka)^2} = \frac{S_{(ka)}}{ka} \cong 5.0\%$$

F. ERROR IN THE SENSITIVITY, M_0

From Chapter II,

$$M_0 = D \left[\frac{\Delta Z \; Z_m}{Z_{rad\,ff}\,(Z_{rad\,ff} + Z_m)} \right]^{1/2} \qquad (2.24)$$

For simplicity, define $[\;] = \dfrac{\Delta Z \; Z_m}{Z_{rad\,ff}\,(Z_{rad\,ff} + Z_m)}$

and $Z_{rad\,ff} = Z_r$.

Differenciating equation 2.24,

$$\delta M_0 = M_0 \frac{\delta D}{D} + \frac{1}{2} D [\;]^{-1/2} \times$$

$$\left[\frac{[\;]\,\delta\Delta Z}{\Delta Z} + \frac{\Delta Z \delta Z_m}{Z_r^2 + Z_m Z_r} - \frac{\Delta Z Z_m}{(Z_r^2 + Z_m Z_r)^2} (2Z_r \delta Z_r + Z_m \delta Z_r + Z_r \delta Z_m) \right]$$

which can be written as

$$\frac{\delta M_0}{M_0} = \frac{\delta D}{D} + \frac{D}{2} \; \frac{\delta\Delta Z}{\Delta Z} + \frac{Z_r\,(\delta Z_m/Z_m) + (2Z_r + Z_m)(\delta Z_r/Z_r)}{(Z_r + Z_m)}$$

but from sections D and E above,

$$\frac{\delta D}{D} < \frac{\delta(ka)}{ka} \quad \text{and} \quad \frac{\delta Z_r}{Z_r} \leqslant 2 \frac{\delta(ka)}{ka}$$

so that

$$\delta M_0 \cong 1 + \frac{D\,(2Z_r + Z_m)}{2(Z_r + Z_m)} \quad \frac{\delta(ka)}{ka} + \frac{D}{2}\;\frac{\delta\Delta Z}{\Delta Z} + \frac{Z_r}{(Z_r + Z_m)} \quad \frac{\delta Z_m}{Z_m}\ .$$

Near resonance take $Z_r \cong Z_m$ and $D < 1$.

Then

$$\frac{S_{Mo}}{M_0} < \left[\left(\frac{2\,S(ka)}{ka}\right)^2 + \left(\frac{S\Delta Z}{2\,\Delta Z}\right)^2 + \left(\frac{S_{Z_m}}{2\,Z_m}\right)^2\right]^{1/2}\ .$$

$$\frac{S_{Mo}}{M_0} \cong \left[(2 \times 5\%)^2 + (0.5 \times 0.4\%)^2 + (0.5 \times 10\%)^2\right]^{1/2}$$

$$\frac{S_{Mo}}{Mo} \cong 11\%$$

Away from resonance let $Z_r \ll Z_m$ and let $(\delta Z_m/Z_m) \sim (\delta Z_r/Z_r)$ and take $D \leqslant 1$, so that

$$\frac{\delta M_0}{M_0} \cong \frac{\delta D}{D} + \frac{D}{2}\;\frac{\delta\Delta Z}{\Delta Z} + \frac{\delta Z_r}{Z_r}$$

$$\cong \frac{2\,\delta(ka)}{ka} + \frac{1}{2}\frac{\delta\Delta Z}{\Delta Z}$$

so

$$\frac{S_{M_0}}{M_0} \equiv \left[\left(\frac{2\, S(ka)}{ka} \right)^2 + \left(\frac{S_{\Delta Z}}{2\, \Delta Z} \right)^2 \right]^{1/2}$$

$$\frac{S_{M_0}}{M_0} \leqslant \left[(2 \times 5\%)^2 + (0.5 \times 0.4\%)^2 \right]^{1/2}$$

$$\frac{S_{M_0}}{M_0} \leqslant 10\% \; .$$

Converting the result to decibels:

Error in dB = 20 log 1 +(S_{M_0}/M_0) ≅ 20 log 1.1 = 0.8 dB

Thus the estimated calibration error is approximately $^+/_-$ 1 dB for all frequencies.

LIST OF REFERENCES

1. Taylor, S. A. , In-Service Sensitivity Measurement Of the Mulloka Sonar Transducer, M. S. Thesis, Naval Postgraduate School, Monterey, California, March 1987.

2. Westbrook, D. E. III, Application of Automated In-Service Performance Monitoring Techniques to the AN/BQQ-5 Spherical Array Transducers, M. S. Thesis, Naval Postgraduate School, Monterey, California, June 1987.

3. Bobber, R. J., Underwater Electroacoustic Measurements, Naval Research Laboratory, Washington, D.C., 1970.

4. Sabin, G. A., "Transducer Calibration by Impedance Measurements" , Journal of Acoustical Society of America, Vol. 28, pp. 705-710, 1956.

5. Schebella, K. J. , Mulloka Self Reciprocity Technique Evaluation, undated report, Advanced Engineering Laboratory, Underwater Systems Development, Defence Research Centre-Salisbury, South Australia.

6. Green, C. E., NUC Sonar Transducer Calibration Facilities, NUC TP 232,Naval Undersea Research and Development Center, San Diego, California, July 1971.

7. Henriquez, T. A. , "Diffraction Constants of Acoustic Transducers", Journal of Acoustical Society of America, Vol. 36, pp. 267-269, 1956.

8. Beranek, L. L. , Acoustic Measurements, John Wiley and Sons, Inc. , 1949.

9. Wilson, O. B. , An Introduction to the Theory and Design of Sonar Transducers, U. S. Government Printing Office, 1985.

10. Hunt, F. V. , Electroacoustics, The Analysis of Transduction and its Historical Background, reprint of the 1954 edition, American Institute of Physics (for the Acoustical Society of America), 1982.

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Thesis
B34035 Bédard
c.1 Reciprocity calibration
of an underwater transducer using the Delta-Z
Method.